Modified Dynamic Programming Method for Reactive Power Compensating Device Allocation

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Abstract – The main topic of this paper refers to reactive power compensating device allocation. By means of modified dynamic programming method, as proposed in this paper, optimal rated power and location of a compensating device are obtained in order to simultaneously satisfy voltage limit constraints, minimize active power loss and avoid voltage collapse in transmission network. In other words, it is necessary to unify all these criteria and find single optimal solution. Software package OPTLOK is developed on the basis of proposed method. At the end, an example given for the realistic transmission network of Croatia is exercised. Croatia, as well as some other South East Europe countries is expected to face serious reactive power compensation problem after the UCTE reconnection that happened recently.

Index Terms - reactive power compensation, voltage profile, voltage collapse, active power loss

I. INTRODUCTION

The main task of each System Operator (SO) is to ensure demanded level of supply quality to its customers. In other words, frequency and voltage profile should be within pre-specified limits at all system nodes with demanded system reliability level. All these criteria should be fulfilled with minimal expenses. Transmission system planning process should include an overview on reactive power compensation problem. Generally, reactive power compensation can be divided into primary, secondary and tertiary regulation. Primary regulation includes performance of high-speed regulating systems such as 1) automatic voltage regulator of synchronous generator excitation systems, 2) on-load tap changing transformers, 3) reactive power compensating devices. Primary regulating devices are usually used for control of smaller variations. Consequently, it is necessary to coordinate reactive power flows from a single hierarchical centre. Secondary regulation aims to change primary regulator input values from a single coordinating centre to reach satisfactory voltage profile in certain part of transmission system. Finally, tertiary regulation assumes coordination of regional secondary regulators. Each SO should have clear rules on primary, secondary and tertiary regulation, especially in deregulated environment where this kind of regulation should be a part of ancillary services market.

Transmission network operation and control can be significantly improved by correct reactive power regulation. Compensating device connection to the power grid influences voltage profile and reactive power flows in the network. Active power loss is also changed. So, reactive power compensation can be analyzed at least from two standpoints: voltage profiles and active power loss. If voltage collapse criterion is added, this problem becomes very complex. Objective function is to find the least cost solution of compensating device connection to transmission grid in order to satisfy voltage profile, minimize active power loss and avoid voltage collapse. Mathematical model presented in this paper comprises all three criteria with regard to operational limits and provides compensating device size and location.

II. MATHEMATICAL BACKGROUND

At the beginning, it is important to point out that this method assumes optimal engagement of all existing reactive power sources (generators, compensating devices), whereas transformer tap-changer positions are optimized with respect to voltage level in the network. It is assumed that the problem could not be solved by using the existing equipment.

Many methods for reactive power compensating device allocation have been developed so far. Existing methods can be divided in several groups as follows:

1) dynamic programming method [1],
2) linear programming method [2], [3], [4],
3) non-linear programming method [5],
4) discrete programming method [6], [7].

The method presented in this paper can be defined as modified dynamic programming method. In classical dynamic programming method [2] after each compensating device switching it is necessary a) to find a new network node with maximal voltage sensitivity coefficient, b) to locate compensating device at that specific node and c) to repeat the procedure till the optimal solution is reached. On the contrary to other methods [2]-[6], programming method proposed in this paper neglects a dynamic component of voltage sensitivity. It calculates the coefficients for many possible system conditions, outages and different system configurations. In that way, average voltage sensitivity coefficients are calculated and single optimal compensating device allocation is obtained for all system states. Also, in the case of more than one candidate for final solution, fuzzy logic approach is used. The main principle used in...
this method is found at the highest voltage change caused by unit reactive power injection. In every single network node (or potential compensating device location) unit reactive power is injected and node voltage changes are recorded. Compensating devices with standard rated power are connected to the most suitable network node until voltage profile is inside the limits. Simultaneously, all voltage constraints (2.1)-(2.3) should be fulfilled.

\[
\begin{align*}
V_j &\leq V_{\text{max}} j \\
V_j &\geq V_{\text{min}} j \\
|\Delta V_j| &\leq \Delta v V_j
\end{align*}
\]

where:
- \(V_j\) voltage at the node \(j; j=1,..,N; j\neq r\),
- \(N\) total number of nodes,
- \(r\) slack bus,
- \(V_{\text{max}}\) upper voltage limit,
- \(V_{\text{min}}\) lower voltage limit,
- \(\Delta V\) absolute voltage value growth with respect to base case value,
- \(\Delta v\) permitted relative voltage growth (usually set to 3\%) defined to avoid extreme voltage changes.

If some of voltage constraints (minimum, maximum allowed value, absolute voltage value growth) cannot be fulfilled at specified network node, next potential node with the highest average voltage sensitivity coefficient is taken into account. The procedure is repeated till all constraints are fulfilled (optimal solution) or all potential nodes are used (sub-optimal solution). If compensating device (with rated power \(Q\)) is connected to the node \(k\), injected node current is:

\[
\Delta I_k = \frac{\pm jQ}{V_k}
\]  

where:
- \(\Delta I_k\) current injected in the node \(k\),
- \(V_k\) node voltage,
- \(Q\) injected reactive power

According to (2.4) voltage increase at the node \(i\) will be:

\[\Delta V_i = Z_{ii} \cdot \Delta I_k = Z_{ii} \left( \frac{jQ}{V_k} \right)\]  

Conventionally, capacitive reactive power is negative, while inductive reactive power is positive. If current \(\Delta I_k\) is injected only at the node \(k\), matrix relation is defined as follows:

\[
\begin{bmatrix}
\Delta V_i \\
\vdots \\
\Delta V_k \\
\vdots \\
\Delta V_N
\end{bmatrix} =
\begin{bmatrix}
Z_{i1} & \ldots & Z_{ik} & \ldots & Z_{iN} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{k1} & \ldots & Z_{kk} & \ldots & Z_{kN} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{N1} & \ldots & Z_{Nk} & \ldots & Z_{NN}
\end{bmatrix}
\begin{bmatrix}
\Delta I_k \\
\vdots \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

\[
\Delta V_i = Z_{ii} \cdot \Delta I_k = Z_{ii} \left( \frac{jQ}{V_k} \right)
\]  

Current \(\Delta I_k\) injected at the node \(k\) causes response at all other nodes of the network. Sum of responses from all nodes with out-of-limit voltages will give value called Voltage Sensitivity Coefficient (VSC). It is defined as sum of absolute voltage value changes at all nodes as a consequence of unit reactive power injection at the node \(k\):

\[
VSC_k = \sum_{i\in\Pi} |\Delta V_i| = \sum_{i\in\Pi} \left| Z_{i,k} \left( \frac{jQ}{V_k} \right) \right| 
\]

where:
- \(\Pi\) set of all nodes with voltage out of limits
- \(V_i \geq V_{\text{max}} i\) or \(V_i \leq V_{\text{min}} i\)

This value presents only one characteristic system condition. If there are \(M_s\) different system states depending on load level, hydrology etc., for every single system state \(i=1,..,M_s\) the voltage sensitivity coefficient at the node \(k\) is given as follows:

\[
VSC_{k,i} = \left( \sum_{i\in\Pi} |\Delta V_i| \right) = \sum_{i\in\Pi} \left| Z_{i,k} \left( \frac{jQ}{V_k} \right) \right| 
\]

Every system state \((i=1,..,M_s)\) is characterized by its probability and voltage weight factor \((w_i)\). No matter of system state probability, there are different voltage value violations. Voltage weight factor \((w_i)\) is defined as the difference between voltage limit and existing voltage value at the voltage violated nodes. Finally, by using all system conditions, respective probabilities and voltage weighting factors, the average voltage sensitivity coefficient can be derived as follows:

\[
VSC_k = \frac{1}{M_s} \sum_{i=1}^{M_s} \left( p_i \cdot w_i \cdot \sum_{l\in\Pi} \left| Z_{l,k} \left( \frac{jQ}{V_k} \right) \right| \right)
\]

where:
- \(p_i\) system condition probability.

III. CRITERION ON VOLTAGE LIMITS

The procedure for optimal compensating device allocation with respect to voltage constraints is defined in 7 steps:

1) At the node with the highest VSC value install compensating device with the lowest standard rated power (the cheapest device); perform load flow and voltage analysis;

2) Check voltage constraints at all nodes as follows:

a) in the case of inductive compensating device:

\[
V_j \geq V_{\text{max}} j \\
|\Delta V_j| \leq \Delta v V_j
\]

b) in the case of capacitive compensating device:

\[
V_j \leq V_{\text{max}} j \\
|\Delta V_j| \leq \Delta v V_j
\]

\(j=1, 2,..,N; j\neq r\).
3) If one or more of the constraints is not fulfilled, the analyzed node is exempted from the potential node list.
4) Choose the next node with the highest VSC and return to step 1)
5) If constraints of step 2) are satisfied, check again all voltage values in the network
6) If all voltages are inside the limits, the solution is optimal
7) If not, inject compensating device with higher standard rating power at the most suitable node and return to step 1). If there is no possibility for higher injection, proceed to the next node with the highest VSC and return to step 1). Repeat the procedure till voltage limits are satisfied.

IV. CRITERION ON ACTIVE POWER LOSS MINIMIZATION

Relation between voltage changes and power changes can be obtained by classical matrix approach:

\[ \Delta P = J \Delta U \]  \hspace{1cm} (2.10)

\[ \Delta P = J \Delta U = J^{-1} [\Delta Q] \]  \hspace{1cm} (2.11)

where

- \( J \) Jacobian matrix,
- \( \Delta U \) column matrix presenting difference between actual and calculated voltage, and
- \( \Delta P \) column matrix containing difference between actual and calculated active and reactive power.

The following relation is derived from (2.10):

\[ d(\Delta P) = [JC]T \Delta U = [JC]T [J^{-1} \Delta Q] \]  \hspace{1cm} (2.11)

where

- \( [JC] \) column matrix presenting partial derivation of active power over voltage and angle, and
- \( \Delta P \) total active power loss.

In (2.11), the matrix \( [L] \) is defined as

\[ [L] = [JC]T [J^{-1}] \]  \hspace{1cm} (2.12)

and has a norm 2N-2Mr, where Mr is a total number of slack busses. First N-Mr matrix elements define active power loss change with respect to active power injection change. The rest of matrix elements describe correlation between active power loss change and reactive power injection change. In other words, these values present sensitivity coefficients and can be used for the compensating device location ranking with respect to the active power loss minimization criterion. The best location node is characterized by the highest coefficient.

Different system conditions have to be analyzed. Each analyzed system state \( (i=1,2,\ldots,Ms) \) is characterized by specific probability \( (p_i; \sum p_i=1) \) and specific matrix \( [L_i] \), where \( i=1,2,\ldots,Ms \). Average sensitivity matrix is defined as a sum of sensitivity matrices and respecting probabilities

\[ [L] = \sum_{i=1}^{Ms} p_i[L_i] \]  \hspace{1cm} (2.13)

On the basis of relation (2.13) it is possible to rank nodes (except the slack one) according to favorableness for compensating device location. Accordingly, only potential locations are conducted which improves calculation speed.

Standard power rating of compensating device is supposed to be injected at potential location nodes. With its operation, active power loss will be changed. Analogue to voltage limit criterion explained in previous chapter, the most acceptable type of compensating device is obtained and incorporated at the most suitable node. If it is not possible to realize compensating device implementation at optimal location, the next possible location with the highest sensitivity coefficient is accepted as sub-optimal solution.

V. THE CRITERION OF VOLTAGE COLLAPSE

Mathematical background for this kind of analysis is based on the matrix \( [D] \) obtained from Jacobian matrix

\[ [D][\Delta V] = [\Delta Q] \]  \hspace{1cm} (2.14)

Dimension of the matrix \( [D] \) is \( M \), where \( M \) is the total number of buses at potential locations \( M \leq (N-Mr) \). Linear operator \( [D] \) transforms voltage vector \( [\Delta V] \) in vector \( [\Delta Q] \). Moreover, the procedure seeks for unknown scalar value \( \lambda \) represented as:

\[ [D][\Delta V] = \lambda \cdot [\Delta V] \]  \hspace{1cm} (2.15)

Relation (2.15) can be rewritten as follows:

\[ ([D]-\lambda \cdot [I]) \cdot [\Delta V] = 0 \]  \hspace{1cm} (2.16)

where:

- \([I]\) unit matrix,
- \([0]\) null vector.

This assumes \( M \) homogeneous linear equations system with \( M \) variables, having non-trivial solutions only if the determinant is equal to zero:

\[ |D-\lambda \cdot [I]| = 0 \]  \hspace{1cm} (2.17)

Relation (2.17) can also be written as follows:

\[ p(\lambda) = |\lambda \cdot [I] - D| \]  \hspace{1cm} (2.18)

where:

- \( p(\lambda) \) characteristic \( [D] \) matrix polynom.

Equation (2.18) can also be written as:

\[ p(\lambda) = \prod_{i=1}^{M} (\lambda - \lambda_i) \]  \hspace{1cm} (2.19)

where:

- \( \lambda_i \) characteristic polynomial solutions \( (i=1,2,\ldots,M) \), or matrix \( [D] \) eigenvalues.

Adequate mathematical analysis proves that matrix eigenvalues \( \lambda \) are related to diagonal matrix elements \( [8] \). The absolute values of eigenvalues are voltage collapse proximity indicators. If these values indicate voltage collapse proximity, the bifurcation point distance for each node is evaluated by means of \( \varepsilon_j \) value \( (j=1,2,\ldots,M) \), according to [11].

Sensitivity matrix (vector) $[c]$ can be formed of individual $c$ values. Each system state is characterized by its probability $p_i$. The average sensitivity matrix is defined as the sum of products of system condition probabilities and eigenvalues in the following form:

$$[c] = \sum_{i=1}^{Ms} p_i [c_i]$$  \hspace{1cm} (2.20)

VI. FINAL SOLUTION DETERMINATION

After three independent analyses it is necessary to find the unique solution that will optimize all three criteria in short and long term time horizon. Depending on realistic system operational characteristics and operating experience, each criterion is associated its weight factor ($w$). Weight factors can not be proved by experiment, but by subjective planner’s assessment. Final solution is defined as the sum of the following products:

$$[f] = w_{voltage} [VSC] + w_{loss} [f] + w_{collapse} [f]$$  \hspace{1cm} (2.21)

The most suitable location node for compensating device installation is defined by the highest value of the matrix $[f]$. Relation for final compensating device rated power determination is set as follows:

$$F(i, j) = w_{voltage} [VSC(i)] + \frac{1}{\Delta V(i, j) + 1} + w_{loss} \cdot L(i) \cdot \Delta P_{loss}(i, j) + 1 + w_{collapse} \cdot \epsilon(i)$$  \hspace{1cm} (2.22)

where:

- $\Delta V(i, j)$: sum of absolute value of voltage deviations (difference between limit and actual value) after compensating device “$j$” allocation at the node “$i$”,
- $\Delta P_{load}(i, j)$: total active power loss reduction after compensating device “$j$” allocation at the node “$i$”,
- $M$: total number of potential compensating device locations,
- Mns: total number of standard compensating device installed capacities.

It is necessary to use unit or norm values in relation (2.22) to avoid dominance of one criterion with respect to another. Finally, it is possible to form global sensitivity matrix with rows defining nodes and columns defining standard rated power variants.

If there are more nodes with similar $f$ values, fuzzy logic approach can be used to make precise node graduation. Each node candidate is assigned with a fuzzy membership value obtained by using AND logic function for three independent criterion membership functions.

VII. EXAMPLE

Previously described modified dynamic programming method is used as a basis for appropriate software package development (named Optlok). This software package is used to determine optimal location and size of the compensation device(s) in Croatian transmission network for short-term and mid-term development [9].

Time horizon 2005 - 2010 is observed. Croatian transmission system has 400 kV, 220 kV and 110 kV voltage levels with longitudinal structure of EHV lines (Fig. 1). Production facilities are mostly connected to 220 kV and 110 kV network, remote from largest consumer centers. Large amount of hydro power plant capacity (nearly half of total domestic production) makes Croatian power system significantly dependent on hydrological conditions. Large differences between maximum and minimum daily load (1:0.5; due to collapse of industrial customers), large difference between maximum and minimum annual load (1:0.35) and dependence on the hydrological conditions cause variable transmission network loadings.

Consequence of such power system characteristics is referred to occasional voltage problems, stressed with the fact that voltage/reactive power control possibilities are very limited due to the lack of appropriate compensating devices. Only synchronous generators and tap changing transformers, together with only one capacitor bank (3x16 MVA) and reactor (2x50 MVA) connected to 110 kV voltage level are used to control voltages and reactive power flows. Additional problem is made due to the lack of adequate financial compensation for reactive power production, so power plant operators usually try to avoid under excited operation of the generators. That is the main reason why Croatian power system is exposed to large voltages during summer and spring light loads where voltages in some nodes may rise up to 440 kV and 260 kV (420 kV and 245 kV are maximum permitted values). Moreover, in 1991 UCTE power system was divided in two synchronous zones due to war activities in South East Europe. After 13 years of two existing synchronous zones in this region UCTE reconnection was realized in October 2004 and all neighboring countries are expecting additional voltage problems during light load regimes.

Power flow model used in this paper includes all three transmission voltage levels in Croatia, groups of generators and unit transformers and existing compensation devices. The typical model of Croatia consists of 232 busses, 352 branches, 38 plants, 71 transformers and 133 loads. Loads are modeled at 110 kV voltage level as constant active and reactive power flows. Additional problem is made due to the lack of appropriate compensating devices. Only synchronous generators and tap changing transformers, together with only one capacitor bank (3x16 MVA) and reactor (2x50 MVA) connected to 110 kV voltage level are used to control voltages and reactive power flows. Additional problem is made due to the lack of adequate financial compensation for reactive power production, so power plant operators usually try to avoid under excited operation of the generators. That is the main reason why Croatian power system is exposed to large voltages during summer and spring light loads where voltages in some nodes may rise up to 440 kV and 260 kV (420 kV and 245 kV are maximum permitted values). Moreover, in 1991 UCTE power system was divided in two synchronous zones due to war activities in South East Europe. After 13 years of two existing synchronous zones in this region UCTE reconnection was realized in October 2004 and all neighboring countries are expecting additional voltage problems during light load regimes.

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Many system scenarios are defined (224 in total) concerning observed time horizon (2005 and 2010), consumption (peak and light load), power plant dispatch dependent on hydrological conditions (normal, very dry and very wet hydrology), exchanges with neighboring countries (Hungary, Bosnia and Herzegovina, Serbia), electricity transits for third parties (Italy, Serbia, Bosnia and Herzegovina, Bulgaria) and pumped storage power plants dispatching (out of service, generator, motor,
compensation operational conditions). User defined possibilities (weighting factors) are added to each operating condition based on planners and dispatchers experience.

Possible compensation device locations are set to all 400 kV and 220 kV nodes. Three standard sizes of compensation devices (reactors and capacitors) are analyzed (50, 100 and 150 Mvar). Other sizes and 110 kV nodes are not included in the analysis in order to avoid installation of too many devices in the network which is estimated as economically unacceptable.

Power flow simulations have shown that unacceptable voltage conditions (high voltages) occur at 400 kV voltage level for most analyzed light load scenarios. Voltage conditions in 220 kV and 110 kV voltage levels may be kept under permitted maximum values using automatic or manual tap changing on 400/220 kV, 400/110 kV (manual control) and 220/110 kV (automatic control) transformers. Lines and transformers loading are under permitted values (thermal line rating and nominal transformer power) for all analyzed operating conditions.

Active power loss is within the range of 15 MW to 84 MW. In peak load conditions bus voltage magnitudes are slightly above rated values.

Optimal location and size of compensation device(s) are determined with respect to observed variables individually (voltages, power loss, voltage collapse proximity) for different groups of operating conditions (peak load 2005, peak load 2010, peak load 2005 and 2010, the same for light load, peak and light load 2005, 2010). Analysis of different groups of operating conditions is carried out as to determine if there is a need to install different compensation devices (reactor, capacitor) at the same node which could lead to the justification to use SVC.

The analysis have shown that optimal locations are very different and spread over power system for different observed criteria and groups of operating conditions.

Obtained solutions depend on the chosen system scenarios. Accordingly, list of the first three solutions is given in the following table for the following groups of the system states:

Table 1 Optimal compensating device allocation for different groups of system scenarios

<table>
<thead>
<tr>
<th>System scenarios</th>
<th>Optimal location and rated power of compensating device</th>
<th>Voltage collapse criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak 2005</td>
<td>Meduric 220,C100* Ernostinovo 400,50</td>
<td></td>
</tr>
<tr>
<td>Light 2005</td>
<td>Obrovac 400,L150 Konjsko 400,L50</td>
<td>Zerjavinec 220,50</td>
</tr>
<tr>
<td>Light 2010</td>
<td>Meduric 220, C100 Mraclin 220,50</td>
<td></td>
</tr>
<tr>
<td>Light 2005,2010</td>
<td>Obrovac 220,L150 Tumbri 400,100</td>
<td></td>
</tr>
<tr>
<td>Light 2005,2010</td>
<td>Meduric 220, C100 Mraclin 220,50</td>
<td></td>
</tr>
<tr>
<td>All scenarios</td>
<td>Zerjavinec 220,50 Mraclin 220,50</td>
<td></td>
</tr>
</tbody>
</table>

Optimal location and rated power of compensating device(s) are determined with respect to observed variables individually (voltages, power loss, voltage collapse proximity) for different groups of operating conditions (peak load 2005, peak load 2010, peak load 2005 and 2010, the same for light load, peak and light load 2005, 2010). Analysis of different groups of operating conditions is carried out as to determine if there is a need to install different compensation devices (reactor, capacitor) at the same node which could lead to the justification to use SVC.

This study has resulted with two candidate node for optimal compensating device allocation. Accordingly, standard modified triangle fuzzy membership function for voltage profile is defined. Final optimal compensating device location is 400 kV node Obrovac (connection node of pumped storage power plant “Velebit”) while optimal size of reactor is set to 150 Mvar.

VIII. CONCLUSION

Mathematical method for determination of optimal location and size of compensating devices is described in this paper [10]. Optimization is based on three criteria: voltage profile, active power loss and voltage collapse proximity. Optimal location of compensating device is determined with respect to all three criterions. Mathematical method is used as a basis for software package developed in Energy Institute Hrvoje Pozar, Zagreb. This package is important tool for transmission network planners when conducting voltage/reactive power analyses. Technical aspects of U/Q ancillary services in competitive electricity markets could be perceived using this method and developed software package. For the purpose of official mid and long term development purposes of Croatian power system, optimal location of compensating device is set to 400 kV node Obrovac, while optimal reactor size is set to 150 Mvar.
IX. REFERENCES


X. BIOGRAPHIES

Goran Majstrovic received his B.S., degree from University of Split, Croatia, in 1998. From 1998 he is with the Energy Institute Hrvoje Pozar, Zagreb, working on transmission system planning and analysis and market design. In 2001 he received his M.S. degree from University of Zagreb, Croatia. He is IEEE student member.

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